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X  
A

## Gulf Mathematics Olympiad – 2007

(Organized by the Council of CBSE Affiliated Schools in the Gulf)

Class X

Organizing School: **The Indian School, Bahrain.**

Time Allotted: 3 Hrs

Maximum Marks: 100

### General Instructions:

1. This question paper contains two parts – Part I and Part II.
2. Part I and Part II are given separately and must be answered accordingly.
3. Answer the questions in the space provided for each question.
4. Write the appropriate steps to reach the answer.
5. **Use of calculator is not permitted.**
6. Use the sheets attached at the end for any calculations required.
7. Do not write your name or roll number on any page other than this page.
8. All questions are compulsory.

### Part I

1. Question numbers 1 – 20 are multiple choices, each of which is provided with four alternative answers. Each question carries one mark.  
*For some questions alternate questions are given. Answer one alternative question and the other question should be crossed out by you in the question paper, making it clear as to which alternative question has been answered*
2. Indicate the correct answer in the space provided in the box only

### Part II

1. Question numbers 1 to 6 are of 2 marks.
2. Question numbers 7 to 12 are of 3 marks.
3. Question numbers 13 to 20 are of 4 marks.
4. Question numbers 21 to 23 are of 6 marks.

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Code Number to be allotted by the Regional Organizer

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Total Marks Scored

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SET - A  
PART-1

1. Find the value of  $\alpha$  and  $\beta$  for which the following system of linear equations has infinite number of solutions.

$$2x + 3y = 7$$

$$2\alpha x + (\alpha + \beta)y = 28$$

- a)  $\alpha = 4, \beta = 8$     b)  $\alpha = 8, \beta = 4$     c)  $\alpha = -4, \beta = 8$     d)  $\alpha = 4, \beta = -8$

a

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \frac{2}{2\alpha} = \frac{3}{\alpha + \beta} \quad \alpha = 4$$

$$\frac{2}{8} = \frac{3}{4 + \beta} = \frac{7}{28} \quad 2(\alpha + \beta) = 8 \times 3 \quad \beta = 8$$

$$8 + 2\beta = 24 \quad 2\beta = 16 \quad \beta = 8$$

2. The HCF of the following polynomials

$$12(x^3 + x^2 + x + 1) \text{ and } 18(x^4 - 1) \text{ is}$$

- a)  $6(x+1)^2(x-1)$     b)  $6(x^2+1)(x-1)$     c)  $6(x^2+1)(x+1)$     d)  $6(x+1)^2(x+1)$

c

$$12(x^2(x+1) + (x+1)) \quad | \quad 18(x^2+1)(x^2-1)$$

$$12(x^2+1)(x+1) \quad | \quad 18(x+1)(x-1)(x^2+1)$$

$$6(x^2+1)(x+1)$$

3. The ratio of the seventh to third term of an AP is 12 : 5. Then the ratio of thirteenth to fourth terms is

- a) 10 : 7    b) 7 : 10    c) 3 : 10    d) 10 : 3

d

$$\frac{a+6d}{a+2d} = \frac{12}{5} \quad \frac{a+12d}{a+d} = \frac{6d+11d}{5d+2d} = \frac{17d}{7d} = \frac{17}{7}$$

$$5(a+6d) = 12(a+2d) \quad 5a+30d = 12a+24d \quad 7a = 6d \quad a = \frac{6d}{7}$$

$$\frac{a+13d}{a+d} = \frac{\frac{6d}{7} + 13d}{\frac{6d}{7} + d} = \frac{6d + 91d}{6d + 7d} = \frac{97d}{13d} = \frac{97}{13}$$

4. The value of 'k' for which the quadratic equation

$$(k+4)x^2 + (k+1)x + 1 = 0 \text{ has equal roots is}$$

- a)  $k = 5$  or  $k = -3$     b)  $k = 5$  or  $k = 3$     c)  $k = -5$  or  $k = 3$     d)  $k = -5$  or  $k = -3$

a

$$D = 0$$

$$b^2 - 4ac = 0$$

$$(k+1)^2 - 4(k+4) = 0$$

$$k^2 + 2k + 1 - 4k - 16 = 0$$

$$k^2 - 2k - 15 = 0$$

$$k^2 - 5k + 3k - 15 = 0$$

$$k(k-5) + 3(k-5) = 0 \quad k = 5, -3$$

5. A loan of Rs. 8400 is to be paid in two equal annual installments, the interest being charged at 10% per annum compounded annually. Then each installment is

- a) Rs. 4480    b) Rs. 4840    c) Rs. 4880    d) Rs. 8480

6. The surcharge of 10% is calculated on income tax if the taxable income exceeds  
 a) Rs. 100000      b) Rs. 1000000      c) Rs. 250000      d) Rs. 500000

7. A pole having been broken by wind, the top struck the ground at an angle of  $30^\circ$  and at a distance of 8 m from the foot of the pole. Then the height of the pole before it was broken is

- a)  $2\sqrt{3}$  m      b)  $3\sqrt{8}$  m      c)  $8\sqrt{3}$  m      d)  $8\sqrt{2}$  m



$\tan 30^\circ = \frac{\text{opp}}{\text{adj}}$   
 $\frac{1}{\sqrt{3}} = \frac{\text{opp}}{8}$   
 $\text{opp} = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$   
 $\cos 30^\circ = \frac{\text{adj}}{\text{hyp}}$   
 $\frac{\sqrt{3}}{2} = \frac{8}{\text{hyp}}$   
 $\text{hyp} = \frac{16}{\sqrt{3}} = \frac{16\sqrt{3}}{3}$   
 $\frac{8\sqrt{3}}{3} + \frac{16\sqrt{3}}{3} = \frac{24\sqrt{3}}{3} = 8\sqrt{3}$

8. If  $\sin \theta + \cos \theta = \sqrt{2} \cos \theta$ . Then  $\cot \theta =$

- a)  $\sqrt{2} - 1$       b)  $\frac{1}{\sqrt{2} - 1}$       c)  $\sqrt{2} + 1$       d)  $\frac{1}{\sqrt{2} + 1}$

$\sin \theta = \cos \theta (\sqrt{2} - 1)$

$\frac{\sin \theta}{(\sqrt{2} - 1)} = \cos \theta$

$\frac{\cos \theta}{\sin \theta} = \frac{1}{\sqrt{2} - 1}$

$\cot \theta = \frac{1}{\sqrt{2} - 1}$


9. The value of  $\left(\frac{\tan 20^\circ}{\operatorname{cosec} 70^\circ}\right)^2 + \left(\frac{\cot 20^\circ}{\sec 70^\circ}\right)^2 - \tan 25^\circ \tan 60^\circ \tan 65^\circ$  is

- a)  $1 - \sqrt{3}$       b)  $\sqrt{3} - 1$       c)  $1 + \sqrt{3}$       d) 0

$\frac{\sin 20}{\cos 20} \div \frac{1}{\cos 20} = \sin 20 \cdot \cos 20$   
 $\frac{\cos 20}{\sin 20} \div \frac{1}{\sin 20} = \cos 20 \cdot \sin 20$   
 $\tan 25^\circ \tan 60^\circ \tan 65^\circ = \tan 25^\circ \tan 65^\circ \tan 60^\circ = 1 \cdot \tan 60^\circ = \sqrt{3}$

10. The diameter of the moon is approximately one fourth of the diameter of the earth. Then the ratio of their volumes is

- a) 1 : 64      b) 1 : 16      c) 1 : 4      d) 1 : 8


 $\frac{d_m}{d_e} = \frac{1}{4}$   
 $d_m = \frac{1}{4} d_e$   
 $\frac{d_m}{d_e} = \frac{1}{4}$   
 $\frac{r_m}{r_e} = \frac{1}{4}$   
 $\frac{V_m}{V_e} = \frac{1}{64}$

11. The ratio of the T.S.A of a cube to the T.S.A of another cube whose one side is equal to the diagonal of the face of first cube is

- a) 1 : 8      b) 1 : 4      c) 1 : 2      d) 1 : 6

12. A vessel is in the form of an inverted cone, open at the top and is filled with water up to the top. Its height is 8 cm and radius of the top is 5 cm. When spherical lead shots, each of radius 0.5 cm are dropped in the vessel, one-fourth of the water flows out. Then the number of lead shots dropped in the vessel is

- a) 200      b) 150      c) 250      d) 100

13. A coin is tossed thrice, the probability of getting at least one head is

- a)  $\frac{7}{8}$       b)  $\frac{1}{2}$       c)  $\frac{3}{8}$       d)  $\frac{1}{4}$

14. A card is drawn from a well shuffled deck of playing cards. The probability of getting an ace or a red face card is

- a)  $\frac{1}{13}$       b)  $\frac{5}{26}$       c)  $\frac{3}{26}$       d)  $\frac{4}{13}$

15. The average height of 30 students is 150. It was detected later that one reading of 165 cm was wrongly read as 135 cm. Then the correct mean of the group is

- a) 150      b) 151      c) 152      d) 149

16. The co-ordinates of the point of intersection of medians of triangle ABC, whose vertices are  $(-7,5)$ ,  $(-1,-3)$  and  $(5,7)$  is
- a)  $(1,3)$       b)  $(3,1)$       c)  $(-1,3)$       d)  $(-1,-3)$

17. The three vertices of a parallelogram are  $(a+b, a-b)$ ,  $(2a+b, 2a-b)$  and  $(a-b, a+b)$ . The fourth vertex is
- a)  $(-b, b)$       b)  $(-b, -b)$       c)  $(b, -b)$       d)  $(a, b)$

18. Triangle ABC is right angled at C. If 'p' is the length of the perpendicular from C to AB and  $AB = c$ ,  $BC = a$  and  $CA = b$ , then we have the result
- a)  $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$       b)  $p^2 = \frac{a^2 - b^2}{a^2 b^2}$       c)  $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$       d)  $a^2 + b^2 = p^2$

19. Two men on either side of a temple 75 m high observe the angles of elevation of the temple to be  $30^\circ$  and  $60^\circ$  respectively. The distance between the two men is
- a) 100      b)  $100(1 + \sqrt{3})$       c)  $100\sqrt{3}$       d)  $100(1 - \sqrt{3})$

20. The triangle in which its circum centre lies on one of the sides is
- a) Isosceles triangle      b) Equilateral triangle      c) Right triangle      d) None

**PART II**

1. A takes three hours more than B to walk 30 km, but if A doubles his speed, he is ahead of B by  $1\frac{1}{2}$  hours. Find their speeds of walking.

2. Solve  $\left(\frac{2x}{x-5}\right)^2 + \frac{10x}{x-5} - 24 = 0$

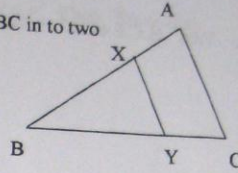
3. An unbiased die is thrown twice. Write the sample space. What is the probability of getting sum of two outcomes in the experiment is a prime number.

4. If  $(3, 2)$ ,  $(-3, 5)$  and  $(x, y)$  are collinear. Show that  $x + 2y - 7 = 0$

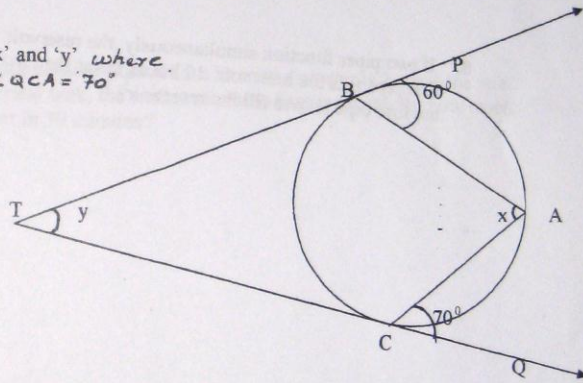
5.

In figure,  $XY \parallel AC$  and  $XY$  divides  $\Delta ABC$  in to two  
Parts equal in area.

Determine  $\frac{AX}{AB}$



6. In figure, find 'x' and 'y' where.  
 $\angle PBA = 60^\circ$ ,  $\angle QCA = 70^\circ$





7. Find H.C.F of  $x^3 + 8$  and  $x^4 + 4x^2 + 16$

8. If two pipes function simultaneously, the reservoir will be filled in 12 hours. If the first pipe fills the reservoir 10 hours faster than the second. How many hours will the first pipe take to fill the reservoir?

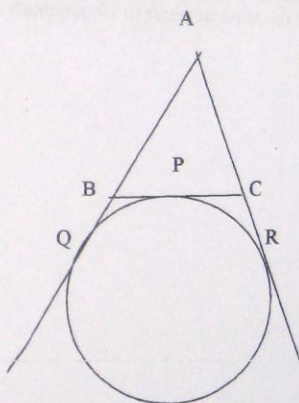
9. If  $a \cot \theta + b \operatorname{cosec} \theta = x^2$  and  $b \cot \theta + a \operatorname{cosec} \theta = y^2$  Then Prove that  $x^4 - y^4 = b^2 - a^2$

10. Water flows out through a circular pipe whose internal diameter is 2 cm at the rate of 6m/sec in to a cylindrical tank, the radius of whose base is 60 cm. By how much will be the level of water in 30 minutes?

11. The expenditure of a household on various heads is given below. Construct a pie-chart for the data.

Head	Food	Rent	Education	Miscellaneous
Percentage of expenditure	$37\frac{1}{2}$	$33\frac{1}{3}$	25	$4\frac{1}{6}$

12. In the following figure  $AB = 5$  cm,  $CR = 3$  cm and  $BC = 4.5$  cm, find the perimeter of  $\Delta ABC$



13. Solve graphically the following system of linear equations in  $x$  and  $y$
- $$4x + 6y = 9$$
- $$2x + 3y = -11$$

14. Simplify  $\frac{a\sqrt{a} + b\sqrt{b}}{(\sqrt{a} + \sqrt{b})(a - b)} + \frac{2\sqrt{b}}{\sqrt{a} + \sqrt{b}} - \frac{\sqrt{ab}}{a - b}$

15. The sum of  $p$  terms of an AP is  $2p^2 + 4p$ . Find the  $n^{\text{th}}$  term.

16. A builder announces the sale of flats for Rs. 30, 00,000 cash or for Rs. 10, 31,600 cash down payment and three equal quarterly installments. If the rate of interest charged is 10 % per annum compounded quarterly. Compute the value of each installment under the installment plan.

17. From the top of a tower the angles of depression of two objects on the same side of the tower are found to be  $\alpha$  and  $\beta$ , ( $\alpha > \beta$ ). If the distance between the object is 'p' meters, show that the height 'h' of the tower is given by 
$$h = \frac{p \tan \alpha \tan \beta}{\tan \alpha - \tan \beta}$$

18. A frustum with upper radius 9 cm made from a solid cone by removing a small cone of height 4cm and base radius 3 cm. Find the volume of the frustum.

21. The annual income of Mrs. Rani, who is a senior citizen is Rs.4,10,000. She donates Rs.30,000 to a temple (50% exemption). She contributes Rs.60,000 towards PPF and pays quarterly premium of Rs.4500 towards life insurance. She also purchases NSC for Rs.30,000. Find the amount she has to pay towards income tax for the financial year.

Use the following slab for finding the tax

Taxable Income	Rate
Up to 1,85,000	Nil
Rs. 1,85,000 to Rs.2,50,000	20% of the amount exceeding Rs.1,85,000
Above Rs.2,50,000	Rs.13000 + 30% of the amount exceeding Rs.2,50,000

Surcharge: 10% of the income tax if the taxable income exceeds Rs.10,00,000.

Educational cess: 2% of the amount of the tax payable.

Concession for savings: Notified savings (PF, LIC, PPF, NSC, Mutual funds etc.) Up to a maximum of Rs.1,00,000 are exempted from the income.



19. Find the value of 'P' if the mean of the following distribution is 20.

X	15	17	19	20 + P	23
F	2	3	4	5P	6

20. In an equilateral triangle, prove that the centroid and the circum- centre coincide.

22. If a line touches a circle and from the point of contact a chord is drawn so that the angles which this chord makes with the given line are equal respectively to the angles formed in the corresponding alternate segments- Prove.

23. Construct a cyclic quadrilateral ABCD where  $AB = 3$  cm,  $BC = 6$  cm,  $CA = 4$  cm and  $AD = 2$  cm. Also construct a quadrilateral similar to ABCD whose sides are  $\frac{3}{2}$  times the corresponding sides of ABCD.